Experimental test of coherent betatron resonance excitations

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(Received 21 July 1997)

Sustained coherent betatron oscillations have been produced in the Brookhaven National Laboratory Alternating Gradient Synchrotron (AGS) using a high-frequency oscillating dipole field operating very near a subharmonic of the betatron tune. The betatron oscillations were adiabatically induced and manipulated in a controlled fashion so that the transverse emittance of the particle beam was preserved. This procedure will be used to induce spin flip in the AGS during the crossing of depolarizing resonances during polarized beam operation. It may also find applications toward measuring nonlinear parameters of transverse motion in synchrotrons and storage rings. [S1063-651X(97)07011-6]

PACS number(s): 41.85.−p, 29.27.−a

I. INTRODUCTION

Coherent betatron oscillations are useful for beam manipulations and beam diagnostics procedures. One application would be in the acceleration of polarized beams in a synchrotron. To avoid depolarization from spin resonances, one either crosses through the resonances quickly or, if the resonances are strong enough, one passes through slowly, which results in an adiabatic spin flip. By inducing a vertical coherent betatron oscillation, it is possible to increase the resonance strength of all particles to the point where this second condition is met with the normal acceleration rate of the synchrotron.

In this and other applications it is important to control and preserve the particle beam emittance. If a betatron oscillation is induced using a pulsed magnet or in some other nonadiabatic manipulation, then the beam will filament in phase space due to guide field nonlinearities inherent in the accelerator and the effective emittance of the beam will increase. To introduce a controlled oscillation adiabatically, a high-frequency dipole magnet can be slowly energized to its final field amplitude and likewise slowly de-energized. This can be achieved with a magnet system in which one can control the field amplitude as well as the frequency of the ac device during the operation.

This can easily be understood by studying the motion in a phase-space frame that is rotating at the modulation frequency that allows for identifying the fixed-point condition. Consider a particle influenced by a single horizontally oriented ac dipole field of length ℓ at a location in the accelerator where the vertical betatron amplitude function is β_z. The field oscillates according to $B_x = B_m \cos 2\pi\nu_m n$, where B_m is the amplitude of the ac dipole field, ν_m is the modulation tune defined as the oscillating frequency divided by the accelerator's revolution frequency, and n is the number of revolutions about the accelerator. Figure 1 shows the phase-space motion in a phase-space frame rotating at the frequency $2\pi\nu_m$. In this frame a particle's phase-space vector rotates through an angle $2\pi\delta$ each turn, with

$$\delta \equiv \nu_z - (k - \nu_m), \tag{1}$$

where k is an integer and ν_z is the vertical betatron tune. The ac dipole field deflects the trajectory through an angle $\theta = B_x \ell / B\rho$, where $B\rho$ is the "magnetic rigidity" (momentum per charge). Since the dipole field is oscillating at the same frequency as the rotating frame, the time average change of Z' in this frame is just half the amplitude of the modulated angular deflection

$$\langle Z' \rangle = \frac{1}{2}(B_m \ell)/B\rho. \tag{2}$$

Therefore, there will be a fixed point in this frame a distance $Z_{\rm coh}$ from the origin given by

$$2\pi\delta Z_{\rm coh} = \frac{1}{2}\beta_z \frac{B_m \ell}{B\rho} \tag{3}$$

or

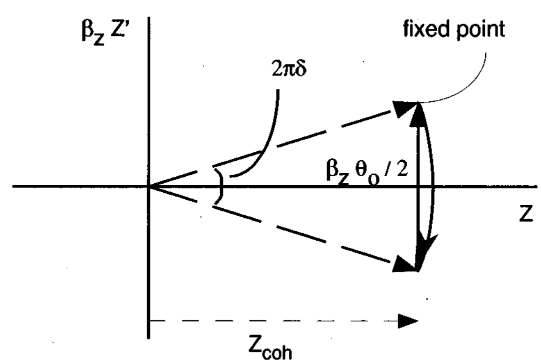

FIG. 1. Derivation of coherent betatron oscillation amplitude in a phase-space frame rotating at the modulation frequency. Each modulation period, the phase-space vector rotates through an angle $2\pi\delta$ and is then given an effective kick by the dipole field by an average amount $\beta_z \Delta Z' = \frac{1}{2}\beta_z\theta_0 \equiv \frac{1}{2}\beta_z B_m \ell / B\rho$.

$$Z_{\rm coh}=\frac{B_m \ell}{4\pi(B\rho)\delta}\,\beta_z\,. \tag{4}$$

Alternatively, this relationship can be obtained by looking at the equation of motion in the vertical plane, namely,

$$z''+K_z(s)z=-\frac{B_m(s)}{B\rho}\cos\,\nu_m\phi(s), \tag{5}$$

where z is the vertical betatron coordinate, the prime corresponds to the derivative with respect to the longitudinal coordinate s, $K_z=(\partial B_x/\partial z)/B\rho$, and $\phi=s/R$, where R is the average radius of the accelerator. The corresponding Hamiltonian is of the form

$$H=\frac{1}{2}z'^2+\frac{1}{2}K_z z^2+\frac{B_m}{B\rho}\,z\,\cos\,\nu_m\phi. \tag{6}$$

By doing several canonical transformations including a transformation to the resonant frame, the final Hamiltonian becomes

$$H(\psi,J)=\delta J+\frac{1}{2}C\sqrt{2J}\,\cos\,\psi, \tag{7}$$

where $J\equiv[z^2+(\alpha_z z+\beta_z z')^2]/2\beta_z$ and ψ, the phase angle in the rotating frame, are the conjugate action-angle phase-space coordinates. As usual, $\alpha=-\frac{1}{2}\beta'$. Additionally,

$$C\equiv\frac{B_m \ell}{2\pi(B\rho)}\,\sqrt{\beta_z}. \tag{8}$$

The fixed points in the resonant frame are obtained from

$$\dot{J}=-\frac{\partial H}{\partial\psi}=0,\quad \dot{\psi}=\frac{\partial H}{\partial J}=0. \tag{9}$$

Equation (9) gives $\psi_{\rm FP}=0$ or π, for which

$$\delta\pm\frac{1}{2}C(2J)^{-1/2}=0\;\rightarrow\;J=\frac{1}{2}\left(\frac{C}{2\delta}\right)^2, \tag{10}$$

where the plus sign corresponds to $\psi_{\rm FP}=0$ and the minus sign to $\psi_{\rm FP}=\pi$. There is only one fixed point since the fixed-point amplitude corresponding to $\psi_{\rm FP}=\pi$ is the negative of the fixed-point amplitude corresponding to $\psi_{\rm FP}=0$. From Eq. (10), the coherent betatron amplitude is then found to be the same as that given by Eq. (4). We see that the amplitude of the ac dipole-induced coherent betatron oscillation is proportional to the maximum dipole field strength and inversely proportional to the distance from the resonant tune.

Effect of detuning

In the above discussion, it has been assumed that the motion could be described as a result of an oscillating field acting on a particle in concert with linear restoring forces. A more realistic situation is one in which the restoring forces are linear to a high degree, but with nonlinear terms present as well. To the next highest order, these nonlinear restoring forces will generate an amplitude-dependent betatron tune. This adds an additional term to the Hamiltonian in Eq. (7), which becomes

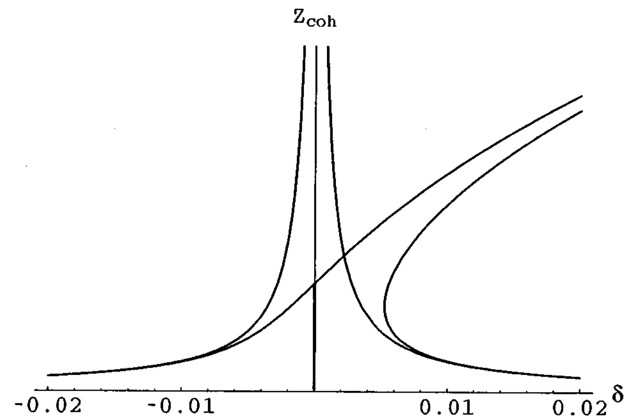

FIG. 2. Distance of fixed points from reference orbit as a function of δ. The ''upright'' curves are for a detuning parameter of zero and the ''slanted'' curves are for an arbitrary $\alpha_{zz}>0$.

$$H(\psi,J)=\delta J+\frac{1}{2}\alpha_{zz}J^2+\frac{1}{2}C\sqrt{2J}\,\cos\,\psi. \tag{11}$$

The betatron tune is thus dependent upon the betatron oscillation amplitude a according to $\nu_z=\nu_0+(\alpha_{zz}/2\beta_z)a^2$. Solutions for the fixed points for this Hamiltonian are shown in Fig. 2. One sees that above a certain bifurcation tune there are actually three solutions, two of which are stable [1]. Thus measurements of the asymmetry in curves of $Z_{\rm coh}$ versus δ could be used in order to determine the amount of transverse detuning present in an accelerator. Similar bifurcations of amplitude response in modulated longitudinal systems have been observed [2] and studied [3,4] elsewhere. Similar bifurcation due to synchrobetatron resonances has been observed [5].

II. EXPERIMENTAL TESTS IN THE AGS

To investigate the feasibility of this procedure, tests were performed in the Brookhaven Alternating Gradient Synchrotron (AGS) during a recent heavy ion physics run. A fast dipole kicker magnet exists in the AGS that is normally pulsed to generate small vertical, short-lived coherent oscillations for measuring the betatron tune. The value of the vertical betatron function at the magnet's location is calculated to be about 16.6 m. For purposes of the experiment, the magnet was disconnected from its normal pulse network and connected as part of a resonant circuit. Measurements of the magnet and its power supply circuit indicated a system resonant frequency of about 70.8 kHz. The current amplitude drops by a factor of 2 when the driving frequency is ± 10 kHz away. This corresponds to a range in ν_m of about 0.21–0.28 for this experiment. The maximum field strength of the magnet with this system is 7.5 G m.

A. Experimental setup

For the beam tests, the power supply circuit for this device was modified to include function generators to control the magnet current. A function generator was used to vary the envelope of the dipole strength amplitude as a function of time. The ac dipole was linearly ramped up in about 3 ms, kept at full amplitude for about 4 ms, and then ramped back to zero in another 3 ms. By adjusting the leading edge, falling edge, and frequency of this function generator's output, the system's ramp-up time, ramp-down time, and the total excitation time were controlled.

During acceleration in the AGS, the revolution frequency changes as the particle momentum increases. Thus, to maintain the ac dipole's modulation tune with respect to the betatron tune, the modulation frequency must sweep. A separate arbitrary function generator was used to generate the sine wave for the ac dipole. The AGS beam rf signal was used as the clock so that the modulation tune was kept constant throughout the acceleration. Multiplying the arbitrary function generator's output with the ac dipole amplitude envelope signal, an amplitude modulated sine wave with a fixed tune was obtained. This allowed the distance of the two tunes to be maintained during beam acceleration.

The experiment was performed in the AGS with Au^{+77} beam. The revolution frequency of the gold beam sweeps from about 280 kHz to about 290 kHz during the experiment. By setting the vertical betatron tune to roughly 8.75, the tune distance from the nearest integer would correspond to a dipole magnet system resonant frequency of $0.25\times290\ \text{kHz}\approx73\ \text{kHz}$. The working momentum for the experiment was therefore about 1.1 GeV/c per nucleon.

The normalized emittance (95%) of the Au^{+77} beam was about 5π mm mrad and the rms beam size at this energy is about 3.9 mm at a maximum amplitude function location. The AGS beam pipe is a $6\times3\ \text{in.}^2$ ellipse. However, the effective vertical aperture in the accelerator is only about 2.8 in. Assuming zero tune spread in the beam and no orbit distortion, the maximum coherent amplitude that could be sustained within the vacuum chamber without beam loss would be about 30 mm.

Transverse beam position measurements were obtained with a pick-up electrode (PUE) located 4.5 betatron oscillations downstream of the ac dipole, where the vertical amplitude function is 15.4 m. A software program was written to remotely operate the oscilloscope, acquire data, calculate the ratio of the difference and sum of the two plates' signals, and then save the data into a text file for future analysis.

The beam transverse profile was measured using the existing AGS ionization profile monitor (IPM) system. This system, with an integrating time of about 5 ms, scanned the beam profile from about 10 ms before the dipole turns on to about 10 ms after it is turned off. Within a single beam cycle, the minimum time between two successive beam profile measurements for this system is 45 ms. Thus the emittance throughout the ac dipole manipulations has to be acquired on different cycles of the AGS, causing small cycle-to-cycle variations in the beam measurements.

B. Results

The coherence amplitude as a function of distance between the ac dipole modulation tune from the intrinsic betatron tune was measured both during acceleration and at fixed energy in the AGS. The ac dipole modulation tune function was held fixed and the betatron tune was changed to produce different tune separations.

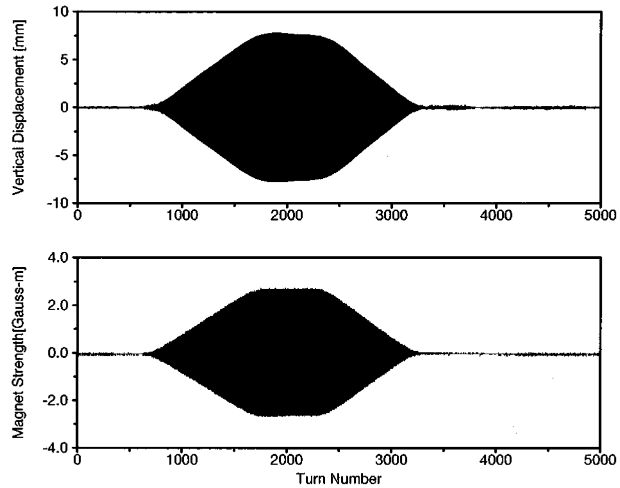

FIG. 3. Transverse displacement (top) and the ac dipole magnet field amplitude (bottom) as a function of revolutions about the AGS.

A typical example of the readback from the ac dipole current transformer as a function of revolution number is shown in the bottom portion of Fig. 3. Here the full dipole strength is 2.82 G m and its modulation tune is 0.75.

The top part of Fig. 3 is the beam's average vertical displacement sampled turn by turn. It shows that the coherent betatron motion nicely follows the ac dipole field amplitude shown in the bottom portion of the figure. The intrinsic betatron tune is 8.745.

Results from a typical emittance scan is shown in Fig. 4. The top portion of the figure shows the beam size (rms, in mm) versus time, with measurements taken in 1-ms intervals. The bottom part of the figure shows a mountain range plot of the corresponding vertical beam profiles. The broadening of the distribution and narrowing as the ac magnet turns off is evident.

Naturally, the actual beam size does not in fact grow and shrink. The measurement time of the IPM system was 3 ms and hence the beam circles the AGS roughly 900 times during the measurement. Thus the rms beam size shown in the plot is the time-averaged value. The maximum ''measured'' rms beam size is related to the actual rms beam size σ_0 and the oscillation amplitude Z_{coh} generated by the ac dipole by

$$\sigma_{meas}=\sigma_0\sqrt{1+\frac{1}{2}\left(\frac{Z_{coh}}{\sigma_0}\right)^2}. \tag{12}$$

The fact that the measured rms beam size returns to its previous value indicates that the process was indeed adiabatic and that the beam emittance was preserved.

The ratio of the measured coherent amplitude to dipole field amplitude is shown in Fig. 5 as a function of different tune separations between the modulation tune and the betatron tune. During this set of measurements, the momentum was held fixed at 1.1 GeV/c per nucleon. This particular ratio is shown because when the tunes were placed near resonance without reducing the dipole field amplitude, the oscillation amplitude would become large enough to induce beam

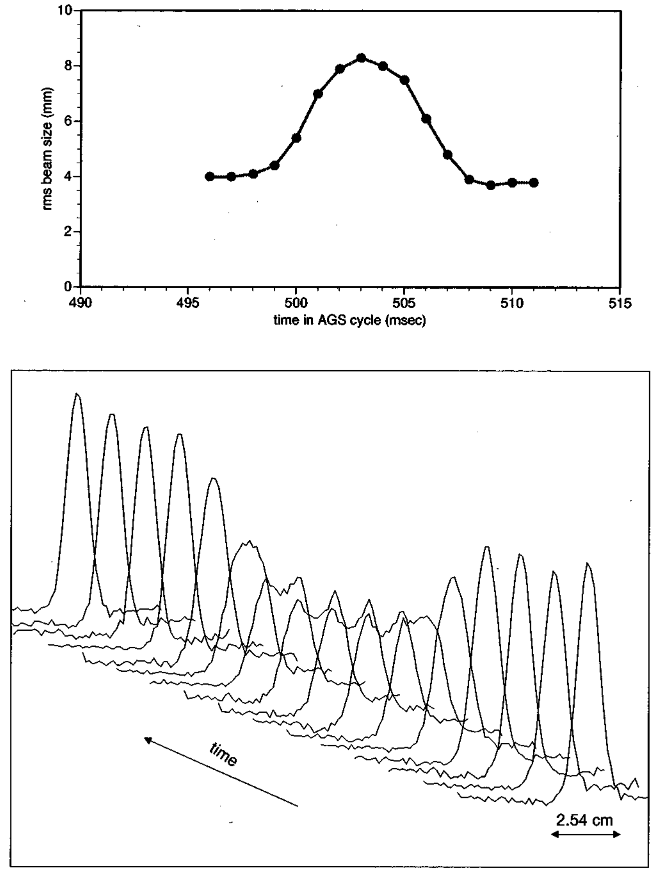

FIG. 4. Measured transverse rms beam size versus time in the AGS cycle (top) and corresponding beam profiles versus time in the AGS cycle (bottom).

loss. Thus only smaller-amplitude oscillations provided meaningful measurements at these tune separations. For a tune separation of 0.01, the largest-amplitude oscillation that could be maintained without significant beam loss was 2.6 times the rms beam size.

A similar set of measurements is depicted in Fig. 6, where in this case the beam is being accelerated at a rate of

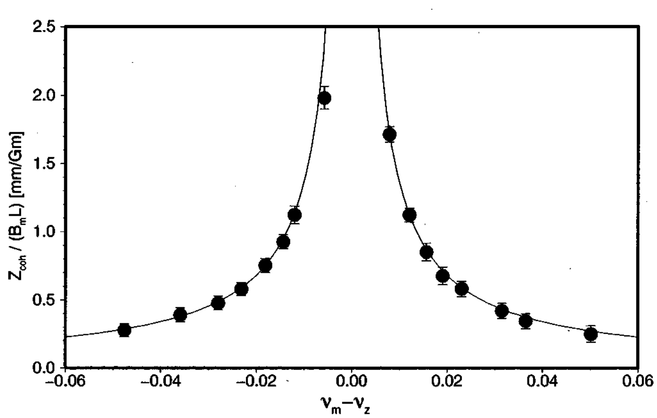

FIG. 5. Normalized coherent betatron oscillation amplitude versus distance from resonance. The beam was held at a fixed energy during these measurements. The oscillation amplitude is normalized by the ac dipole strength. The solid line is the predicted curve using Eq. (4).

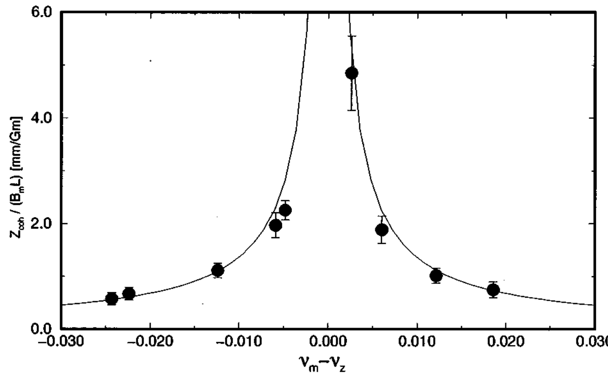

FIG. 6. Normalized coherent betatron oscillation amplitude versus distance from resonance. The beam was being accelerated during these measurements. The oscillation amplitude is normalized by the ac dipole strength. The solid line is the predicted curve using Eq. (4).

$\dot{\gamma} = 2.44\ s^{-1}$. The arbitrary function generator was used to generate the appropriate sine wave with the AGS beam rf signal used as the external clock signal. The solid line is the calculated curve using Eq. (4). Figure 6 shows that by accurately fixing the tune separation, the general features of the ac dipole induced coherent betatron oscillations are indistinguishable between acceleration and storage.

The data can be used, in principle, to determine the amount of nonlinear detuning present in the AGS. However, as seen in Fig. 2, evidence of detuning is better obtained for values of δ much less than 0.01. The smallest value obtained in the experiment was 0.003. The errors in tune measurement (± 0.002) and errors in the analysis of the coherent particle amplitude lead to an estimate of $|\alpha_{zz}| < 750$/m in the AGS. In future tests, attempts will be made to produce stronger detuning using octupole or sextupole magnets and to look for evidence of the nonlinearity in the coherent motion.

C. Effects of tune spread and ripple

As the amplitude of the driven oscillation is inversely proportional to the distance away from the resonant tune, a significant tune spread within the particle beam will generate a range of amplitudes. The beam will thus occupy more of the aperture, which could in turn lead to beam loss. For instance, assume the tune spread of the beam is 0.003 and the driving tune is 0.005 away from the resonance. In order to keep the outer edge of the beam still within the beam pipe, the central coherent oscillation amplitude can only be 40% of what it would be for zero tune spread. So, to make better use of the available aperture and to be able to set the tune closer to resonance, a very small tune spread is desirable.

One of the major sources of tune spread in the AGS is the chromaticity, or the change of betatron tune with momentum. As an example, if the relative momentum spread of the Au^{+77} beam were on the order of 0.001, then to maintain a tune spread of that order the chromaticity needs to be controlled to the level of one unit. In some of the early measurements, the chromaticity was not carefully examined and thus could explain the difficulty in achieving useful measurements with tune separations less than about 0.005. Careful tuning of the chromaticity during the operation of the ac

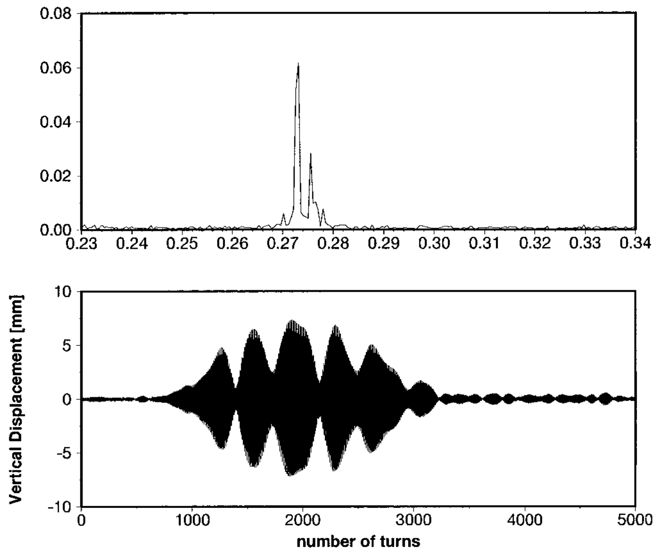

FIG. 7. The lower trace is raw data of transverse oscillations in the presence of power-line ripple. The top trace is the fast Fourier transform of the data, showing the ripple sideband. For this trace, the horizontal axis is the fractional tune. The modulation tune, the large peak, is 0.273.

dipole magnet will be important in future tests. Another method to minimize the betatron amplitude for a given tune spread is to increase the betatron detuning parameter α_{zz}. Using sextupoles and octupoles, the detuning parameter and the nonlinear resonances can be controlled. Octupoles have recently been installed in the AGS, which can be used to perform experiments on the effect of detuning.

During the experiment, problems with power-line ripple were also encountered. In the AGS, an acceleration rate higher than $\dot{\gamma}=2.44\ \mathrm{s}^{-1}$ requires switching between two different bending magnet power supplies to achieve the necessary voltage in the magnet circuit. The more powerful supply introduces 720 Hz power-line ripple to the combined function bending magnets. Thus the rate of change of energy, the betatron tunes, and, to the extent that the rf radial loop can follow, the beam orbit are modulated. The net effect is a modulation of the tune separation δ. Figure 7 shows an example of this ripple effect. The beam coherence signal measured from the PUE is shown in the bottom trace of Fig. 7; a beating structure is evident. The top trace in this figure is the fast Fourier transform of the data, in which ripple sidebands are apparent.

This power supply ripple can be modeled as a betatron tune modulation on the betatron motion. Using the smooth approximation, the equation of motion is given by

$$\ddot{y}+[\nu_0(1+\epsilon\cos\nu_r\theta)]^2 y=a\cos(\nu_m\theta+\chi), \tag{13}$$

where the overdot is the derivative with respect to the orbital angle θ, y is the betatron displacement as a function of θ, ν_0 is the betatron tune, ν_r is the ripple tune, ϵ is the ripple amplitude, ν_m is the modulation tune, χ is an arbitrary phase angle, and a is the modulation amplitude.

To simplify our discussion without loss of generality, we assume $\chi=0$. The forced oscillator equation can be solved by the ansatz

$$y=A\cos\nu_m\theta+\sum_{n=1}^{\infty}B_{n\pm}\cos(\nu_m\pm n\nu_r)\theta. \tag{14}$$

Since ϵ is a small number, we obtain

$$A=\frac{a}{\nu_0^2-\nu_m^2+\dfrac{\nu_0^4\epsilon^2}{(\nu_m-\nu_r)^2-\nu_0^2}+\dfrac{\nu_0^4\epsilon^2}{(\nu_m+\nu_r)^2-\nu_0^2}} \tag{15}$$

and

$$B_{1\pm}=\frac{A\nu_0^2\epsilon}{(\nu_m\pm\nu_r)^2-\nu_0^2}. \tag{16}$$

Similarly, higher-order sidebands can be obtained. Note here that the power supply ripple produces modulated betatron motion with sidebands around the modulation harmonic. It is worth noting that the sideband amplitudes $B_{n\pm}$ are finite even for $\nu_m=\nu_0\pm n\nu_r$. There exists only one dipole mode singularity at $\nu_m\approx\nu_0$.

The initial coherent betatron oscillation in the presence of power supply ripple can decohere to induce emittance growth. Careful control of power supply ripple is important to minimize the emittance growth during this bunched beam manipulation. Plans are being made to reduce power-line ripple in the AGS as well as radial tracking with the rf feedback loop.

III. CONCLUSION

Sustained coherent transverse beam oscillations have been achieved in the AGS using a high-frequency ac driven dipole magnet. The amplitude of the oscillations using the present system has been as large as 2.6 times the rms beam size and the oscillations were held at this level for about 1000 revolutions. By adiabatically increasing and decreasing the dipole field amplitude during this procedure, the transverse emittance of the beam was preserved. To set the scale, had an oscillation of that amplitude been left to oscillate freely, the beam emittance would have increased by a factor of 4 after filamentation. This result provides encouragement that this technique can be used to induce spin flip of polarized proton beams in synchrotrons when crossing strong intrinsic depolarizing resonances during acceleration. Another application of this device in a polarized beam storage ring would be the reversal of the polarization direction of the beam. By slowly sweeping the frequency of the modulated dipole field through the spin precession frequency of the storage ring, an adiabatic spin flip can be induced, thus reversing the polarization direction of each individual particle in the storage ring without increasing the beam emittance. The reversal can be useful for reducing systematic errors in polarized beam experiments. If the tune separation can be further reduced without incurring beam loss, this technique might also be applicable toward measurements of nonlinear detuning in accelerators.

ACKNOWLEDGMENTS

The authors would like to thank L. Ahrens, A. Dunbar, D. Gassner, J. Reich, P. Sampson, R. Sanders, G. Smith, C. Whalen, N. Williams, and K. Zeno for their help during the experiment and A. Yokosawa for fruitful discussions and support. This work was performed under the auspices of the U.S. Department of Energy.

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